

# Design Consideration on Broad-Band *W*-Type Two-Mode Optical Fibers

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**Abstract**—Structural design for broad-band *W*-type two-mode optical fibers is investigated. The optimum parameters are numerically determined as follows: the operating *V*-value with zero group delay time difference  $\Delta\tau$  between the  $LP_{01}$  and  $LP_{11}$  modes is 6.7, the ratio of core radius to inner cladding radius is 0.6, and the index profile parameter is 2.02. Then, the core radius is 12.3  $\mu\text{m}$  for  $\Delta=0.3$  percent at the operating wavelength of 1.3  $\mu\text{m}$ . The *V*-value deviation tolerance from the optimum to maintain  $\Delta\tau$  less than  $\pm 20$  ps/km is 21 percent, which is 20 times larger than that of the earlier design made on two-layer index profile.

## I. INTRODUCTION

IT IS certified from the theoretical and experimental studies [1]–[3] that the two-mode optical fiber provides a large transmission capacity and feasibility of low splice loss. Design principle of the two-mode fiber is that the *V*-value where the group delay times of the two guided modes,  $LP_{01}$  and  $LP_{11}$  modes, coincide is chosen as the operating *V*-value  $V_0$ , and that group delay time difference  $\Delta\tau$  between  $LP_{01}$  and  $LP_{11}$  modes caused by the *V*-value deviation from  $V_0$  is made as small as possible. Here,  $\Delta\tau$  is defined by

$$\Delta\tau = \tau(LP_{11}) - \tau(LP_{01})$$

where  $\tau(LP_{11})$  and  $\tau(LP_{01})$  denote the group delay times of  $LP_{11}$  and  $LP_{01}$  modes, respectively. For the practical use of the two-mode fibers, therefore, it is important to know how to obtain a large tolerance in *V*-value deviation for maintaining  $\Delta\tau$  small over a wide *V*-value region. In the preceding papers [1], [2], designs were made on a two-mode fiber with the two-layer index profile consisting of core and cladding. In this type of index profile,  $V_0$  and the optimum index profile parameter  $\alpha_{\text{opt}}$  have been determined 6.45 and 2.24, respectively, [4] and the tolerance of operating *V*-value deviation is found to be 11 percent for  $\Delta\tau$  less than  $\pm 100$  ps/km.

In order to provide larger tolerance of operating *V*-value deviation, further design consideration is newly made on various index profiles, so-called *W*-type profiles, composed of three layers; core, inner cladding, and outer cladding. This investigation clarifies extremely larger extension of

the small  $\Delta\tau$  region in comparison with that calculated for the two-layer type index profile. For convenience of practical use, the optimum fiber parameters are determined for the *W*-type two-mode fiber from the view point of attaining large deviation tolerances for *V*-value and  $\alpha$ .

## II. $\Delta\tau$ CHARACTERISTICS FOR *W*-TYPE TWO-MODE FIBER

Let us consider *W*-type graded-index fibers consisting of core, inner cladding, and outer cladding. The index profile is expressed by

$$n(r) = \begin{cases} n_1 [1 - 2\Delta\rho(r/a)^\alpha]^{1/2}, & 0 \leq r \leq a \\ n_1 [1 - 2\Delta\rho]^{1/2}, & a \leq r \leq b \\ n_1 [1 - 2\Delta]^{1/2} = n_2, & r > b \end{cases} \quad (1)$$

where  $k$  denotes the wavenumber in vacuum. The parameters chosen are as follows: (a)  $\alpha=3.08$ ,  $\rho=1$ ,  $a=b$ ; (b)  $\alpha=2.04$ ,  $\rho=2$ ,  $a=b$ ; (c)  $\alpha=2.01$ ,  $\rho=2$ ,  $a/b=0.8$ . Calculations are made by numerically solving the vector-wave equation with the matrix method [5]. For simplicity,  $\Delta$  is chosen 0.3 percent in calculations throughout the paper. Cutoff *V*-value  $V_{c2}$  of the  $LP_{21}$  mode for each index profile is also shown in Fig. 2. In case (a), the operating *V*-value

$$V = kan_1\sqrt{2\Delta} \quad (2)$$

where  $k$  denotes the wavenumber in vacuum. The parameters chosen are as follows: (a)  $\alpha=3.08$ ,  $\rho=1$ ,  $a=b$ ; (b)  $\alpha=2.04$ ,  $\rho=2$ ,  $a=b$ ; (c)  $\alpha=2.01$ ,  $\rho=2$ ,  $a/b=0.8$ . Calculations are made by numerically solving the vector-wave equation with the matrix method [5]. For simplicity,  $\Delta$  is chosen 0.3 percent in calculations throughout the paper. Cutoff *V*-value  $V_{c2}$  of the  $LP_{21}$  mode for each index profile is also shown in Fig. 2. In case (a), the operating *V*-value  $V_0$  where  $\Delta\tau=0$  coincides with  $V_{c2}$  for  $\alpha=3.08$  [2]. While, as seen from the results for  $\rho=2$  given by the curves (b) and (c),  $V_0$  is smaller than  $V_{c2}$ . Therefore, it is found that the two-mode *V*-value region is expanded for  $\rho>1$  in comparison with that for  $\rho=1$ . Furthermore, comparing curves (b) and (c) where  $\rho=2$ , small  $\Delta\tau$  characteristics over a wide *V*-value range around  $V_0$  is obtained for  $a/b=0.8$ . As is evident from the curve (b), *V*-value tolerance is enlarged to about 10 times the case with  $\rho=1$  and  $a=b$ . Furthermore, the curve (c) shows that *V*-value toler-

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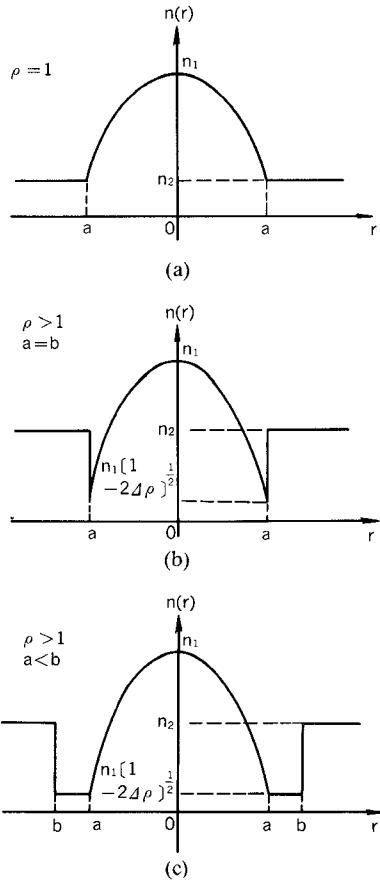


Fig. 1. Three types of index profiles. (a) Two-layer index profile. (b)  $W$ -type index profile with  $\rho > 1$  and  $a = b$ . (c)  $W$ -type index profile with  $\rho > 1$  and  $a < b$ .

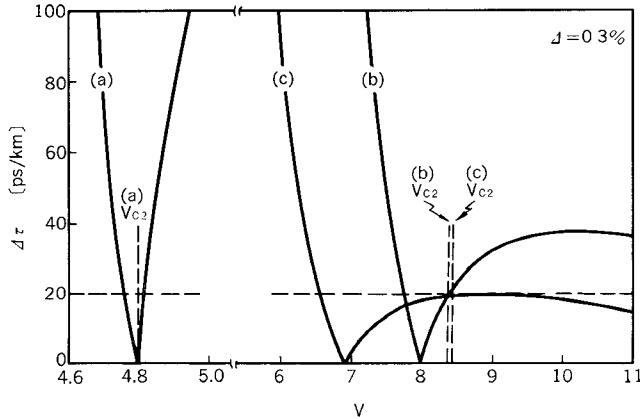


Fig. 2.  $V$ -value dependence of group delay time difference  $\Delta\tau$  between  $LP_{01}$  and  $LP_{11}$  modes, for: (a)  $\alpha = 3.08$ ,  $\rho = 1$ ,  $a = b$ ; (b)  $\alpha = 2.04$ ,  $\rho = 2$ ,  $a = b$ ; and (c)  $\alpha = 2.01$ ,  $\rho = 2$ ,  $a/b = 0.8$ .  $V_{c2}$  denotes the cutoff  $V$ -value of the  $LP_{21}$  mode. Refractive index difference  $\Delta$  is chosen 0.3 percent.

ance is exceedingly extended, compared with the other two index profiles.

### III. PARAMETER TOLERANCE FOR MAINTAINING SMALL $\Delta\tau$ CHARACTERISTICS

Design considerations to optimize the fiber parameters are made on  $W$ -type two-mode fiber. In this section, tolerance of operating  $V$ -value region is numerically evaluated.

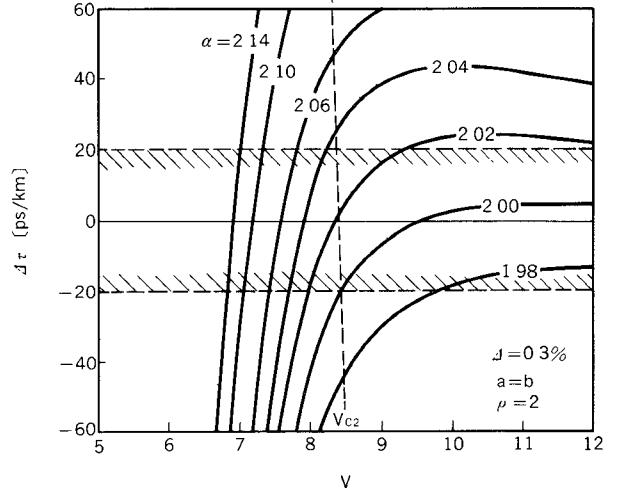


Fig. 3. Relation between  $\Delta\tau$  and  $V$ -value for various values of  $\alpha$  where  $a = b$ .

It is assumed that the allowable limit of  $|\Delta\tau|$  is 20 ps/km, which corresponds approximately to 45 GHz·km in baseband 3-dB bandwidth [4]. Furthermore, operating  $V$ -value region is considered to be restricted below  $V_{c2}$ . Then the tolerance  $B_v$  is defined as

$$B_v = \begin{cases} 2(V_1 - V_2)/(V_1 + V_2), & V_{c2} > V_1 \\ 2(V_{c2} - V_2)/(V_{c2} + V_2), & V_1 > V_{c2} > V_2 \\ 0, & V_2 > V_{c2} \end{cases} \quad (3)$$

for three possible cases, where  $V_1$  and  $V_2$  are  $V$ -values giving  $\Delta\tau = 20$  ps/km and  $-20$  ps/km, respectively. Note that  $V_1$  is always larger than  $V_2$ .

#### A. Index Profile with $\rho > 1$ and $a = b$

Fig. 3 shows the relation between  $\Delta\tau$  and  $V$  for various values of  $\alpha$  where  $\rho = 2$ . The shaded area denotes the  $V$ -value region where  $|\Delta\tau|$  is less than 20 ps/km.  $V_{c2}$  is indicated by the almost vertical dashed line. It is seen from Fig. 3 that there exists the two-mode propagation region ( $V < V_{c2}$ ) with  $|\Delta\tau| < 20$  ps/km for  $\alpha > 2$ , while for  $\alpha \leq 2$  no two-mode propagation region with  $|\Delta\tau| < 20$  ps/km exists. For an index profile having the inner cladding with the refractive-index lower than that of the outer cladding, guided mode power is still confined to some degree in a core region at its cutoff  $V$ -value [6]. Thus the  $LP_{21}$  mode can propagate with relatively low loss near  $V_{c2}$ , compared with the case of  $\rho = 1$ . Therefore, it is considered appropriate at present that  $V_0$  is chosen below  $V_{c2}$  for  $\rho > 1$ , in contrast with the case of  $\rho = 1$  where the operating  $V$ -value region was extended to  $V_0 > V_{c2}$  [4]. In Fig. 4, the operating  $V$ -value tolerance  $B_v$  defined by (3) is plotted against  $\alpha$  for various values of  $\rho$ . It is found that as  $\rho$  increases value of the maximum  $B_v$  becomes large, accompanied with the decrease in the value of  $\alpha$  giving the maximum  $B_v$ . It is noted that  $B_v$  larger than 0.2 is achieved for  $\rho = 2.5$  with  $\alpha \approx 2$ . This value is 20 times larger than that obtained by the previous design with  $\rho = 1$  and  $\alpha = 3.08$  [2]. As shown by the dotted line in Fig. 4, value of 0.5 is

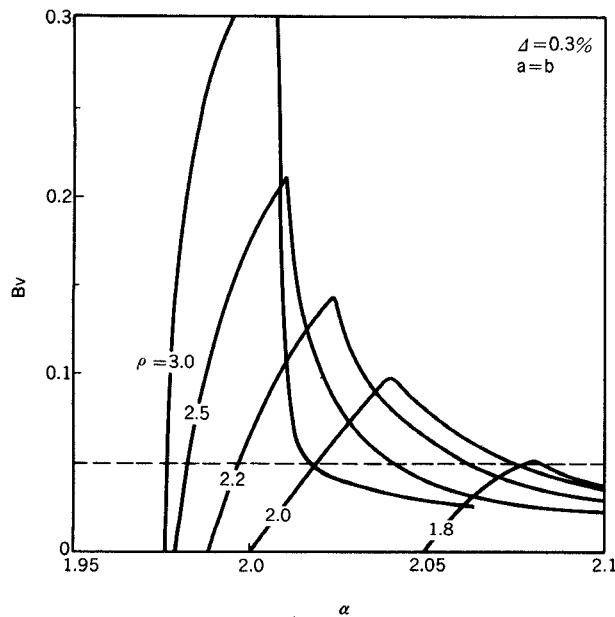


Fig. 4. The operating  $V$ -value tolerance against  $\alpha$  for various values of  $\alpha$  where  $a = b$ .

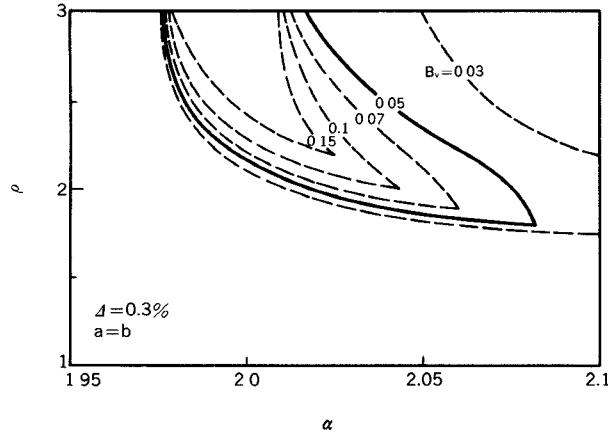


Fig. 5. Combinations of  $\rho$  and  $\alpha$  for various values of  $B_v$  where  $a = b$ .

introduced for the lower limit of the tolerable  $B_v$ . Combinations of  $\rho$  and  $\alpha$  for various values of  $B_v$  are plotted in Fig. 5. It is found from this figure that  $\rho > 1.8$  and  $1.97 < \alpha < 2.08$  are required for satisfying  $B_v > 0.05$  (the region surrounded by the solid curve).

#### B. Index Profile with $\rho > 1$ and $a < b$

Fig. 6 shows the numerical  $\Delta\tau$  against  $V$  for various values of  $\alpha$ , where  $\rho$  and  $a/b$  are assumed 2 and 0.8, respectively. It is found that for  $\alpha$  smaller than 2 two-mode propagation  $V$ -value region with  $|\Delta\tau| < 20$  ps/km still exists due to the presence of inner cladding ( $a/b = 0.8$ ). As a result, remarkable increase in  $B_v$  is observed as shown in Fig. 7. For example, for  $\rho = 2$  the maximum  $B_v$  is 0.25, which is 2.5 times larger than that with  $a = b$ . Fig. 8 shows the combinations of  $\rho$  and  $\alpha$  for  $B_v = 0.05$  with  $a/b$  as a parameter. Within the contours, the condition  $B_v > 0.05$  is satisfied. It is found that as lowering  $a/b$ , the minimum tolerable  $\rho$  for giving  $|\Delta\tau|$  less than 20 ps/km slightly decreases. The values of  $\rho$  are 1.48, 1.35, 1.28 for  $a/b = 0.8$ ,

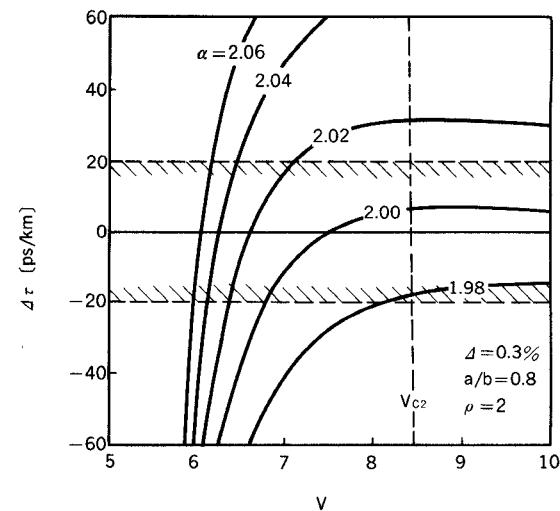


Fig. 6. Relation between  $\Delta\tau$  and  $V$ -value for various values of  $\alpha$  where  $a/b = 0.8$ .

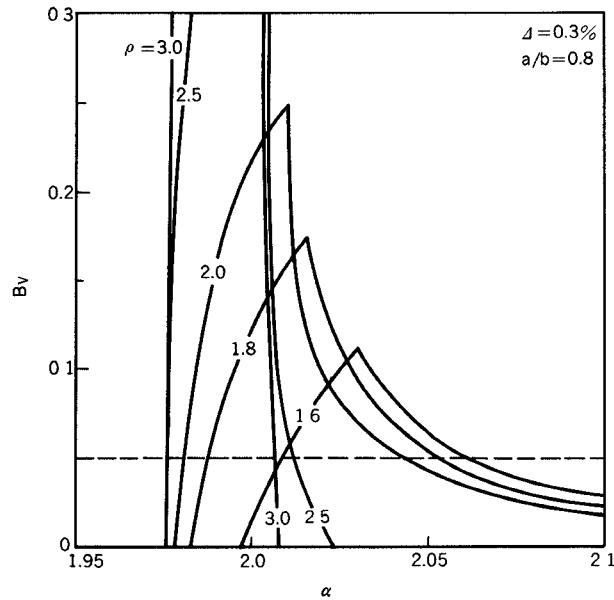


Fig. 7. The operating  $V$ -value tolerance against  $\alpha$  for various values of  $\rho$  where  $a/b = 0.8$ .

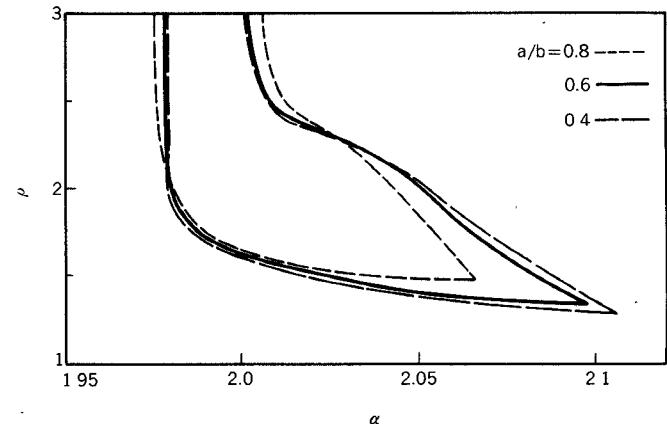


Fig. 8. Combinations of  $\rho$  and  $\alpha$  for various values of  $a/b$  where  $B_v = 0.05$  and  $\Delta = 0.3$  percent.

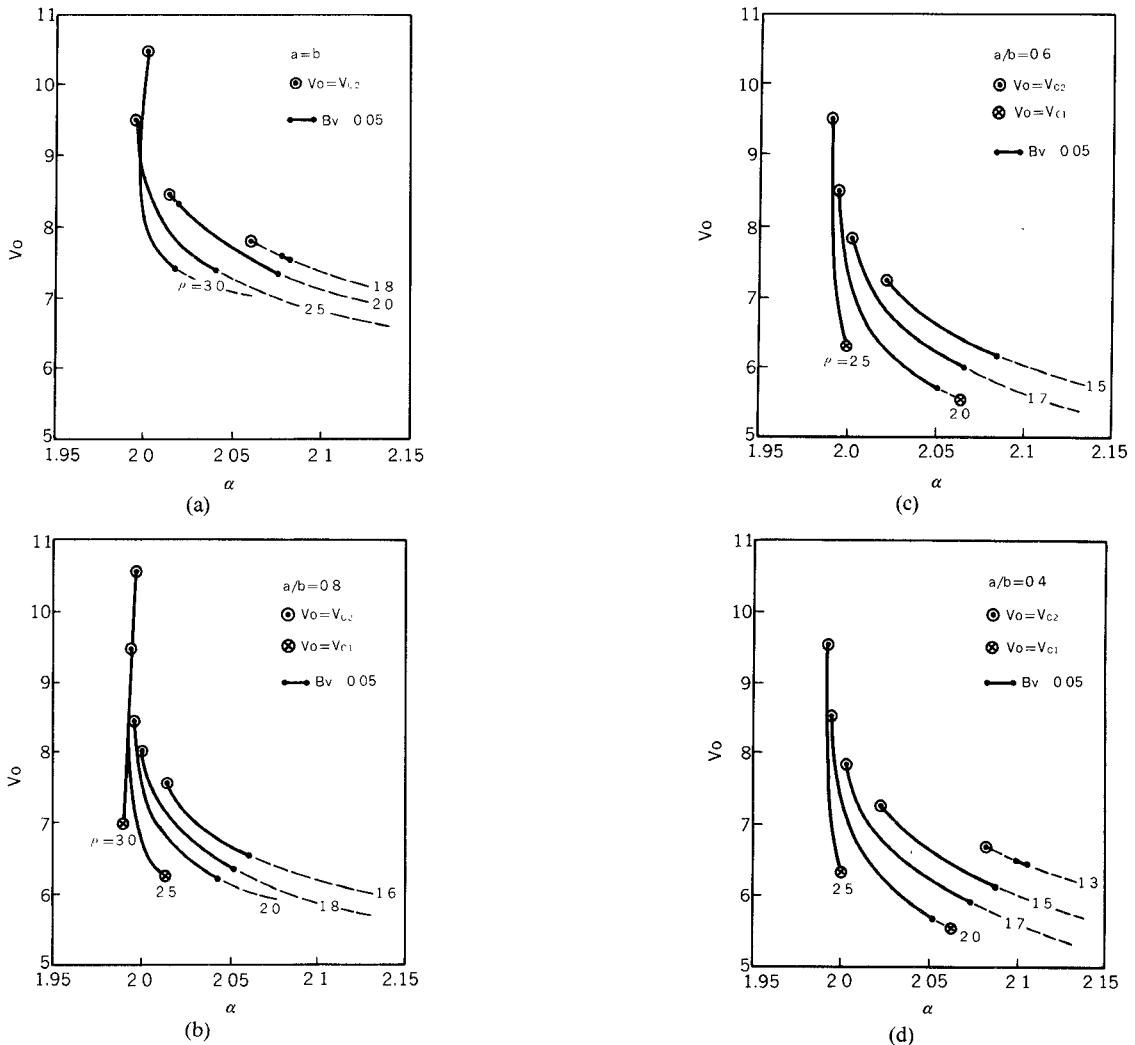


Fig. 9. Dependence of the optimum operating  $V$ -value  $V_0$  where  $\Delta\tau=0$  on  $\alpha$  for various values of  $\rho$ .

0.6, and 0.4, respectively, while  $\rho > 1.8$  in the case of  $a = b$  (Fig. 5).

#### IV. DETERMINATION OF THE OPTIMUM PARAMETERS

It is practically required in view of fabrication ease that the two-mode fiber provides a large  $\alpha$  deviation tolerance as well as the large  $B_v$ . For the convenience of design, the optimum operating  $V$ -value  $V_0$  where  $\Delta\tau=0$  is shown as functions of  $\alpha$  and  $\rho$  in Fig. 9 (a)–(d) for various values of  $a/b$ . The solid curves indicate the useful combinations of  $\alpha$  and  $\rho$ , satisfying the conditions  $V_0 \leq V_{c2}$  and  $B_v \geq 0.05$ , and the dotted curves denote the region where  $B_v < 0.05$ . As seen from the figures,  $V_0$  tends to change rapidly against  $\alpha$  for large  $\rho$ . This feature is unpreferable to obtain a large tolerance in  $\alpha$  deviation, and suggests that the optimum  $\rho$  is less than 2.5. Fig. 10 shows tolerable  $\alpha$  deviation  $\Delta\alpha (= \alpha_{\max} - \alpha_{\min})$  to maintain  $B_v$  larger than 0.05. The optimum  $\rho$  for the maximum  $\Delta\alpha$  is determined around 2 for each value of  $a/b$ , as listed in Table I. Using the values, the optimum  $\alpha$  and  $V_0$  are calculated. The results are summarized in Table I. The value of  $B_v$  against  $\rho_{\text{opt}}$  has the maximum for  $a/b=0.6$ , and decreases for smaller  $a/b$ .

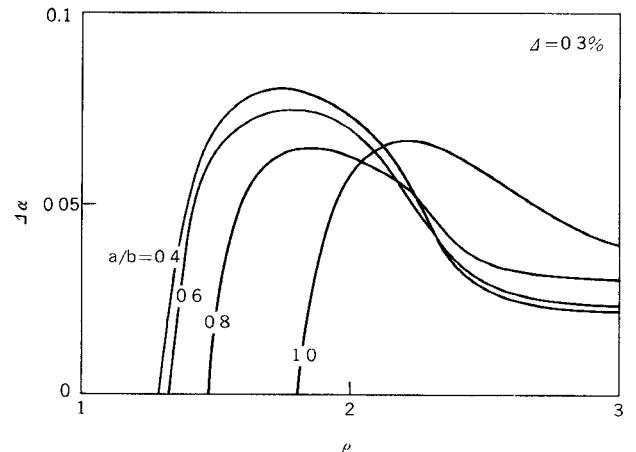


Fig. 10. The tolerable  $\alpha$  deviation  $\Delta\alpha$  as a function of  $\rho$ , to maintain  $B_v$  larger than 0.5.

Furthermore, for small  $a/b$ , the inner cladding radius becomes large, for instance  $b=30 \mu\text{m}$  for  $a/b=0.4$  at 1.3  $\mu\text{m}$ . Such a large  $b$  is not advantageous from the viewpoint of fiber fabrication economy. It is considered from the above discussion that the optimum value of  $a/b$  is chosen

TABLE I  
THE OPTIMUM  $\alpha$  AND  $V_0$  DETERMINED USING THE OPTIMUM  $\rho$  FOR  
VARIOUS VALUES OF  $a/b$

$a/b$	$\rho_{\text{opt}}$	$\alpha_{\text{opt}}$	$V_{\text{opt}}$	$B_v$	$a^*(\mu\text{m})$	$b^*(\mu\text{m})$
1	2.2	2.02	8.1	0.14	14.8	-
0.8	1.8	2.02	7.0	0.18	12.8	16.0
0.6	1.8	2.02	6.7	0.21	12.3	20.5
0.4	1.7	2.02	6.6	0.17	12.1	30.2

\* For  $\lambda = 1.3\mu\text{m}$  and  $\Delta = 0.3\%$

as 0.6. Then, core radius  $a$  and inner cladding radius  $b$  are  $12.3\mu\text{m}$  and  $20.5\mu\text{m}$ , respectively, for  $\lambda = 1.3\mu\text{m}$  and  $\Delta = 0.3$  percent. If both  $\alpha$  and  $V_0$  are set optimum at the wavelength of  $1.3\mu\text{m}$ , tolerable range of wavelength with  $|\Delta\tau| < 20\text{ ps/km}$  extends from  $1.18\mu\text{m}$  to  $1.45\mu\text{m}$  for  $\rho = 1.8$  and  $a/b = 0.6$ . While, the permissible variation in  $\alpha$  ( $\Delta\alpha_{\text{opt}}/\alpha_{\text{opt}}$ ) is 4 percent when the other parameters are fixed at their optimum values.

## V. CONCLUSION

New design of two-mode optical fiber has been presented for *W*-type fibers having wide tolerances of  $V$ -value and  $\alpha$  with group delay time difference between the  $LP_{01}$  and  $LP_{11}$  modes less than  $\pm 20\text{ ps/km}$ . The optimum values of  $\rho$  are found to change with the variation of inner cladding thickness. While it is interesting that the optimum  $\alpha$  is fixed at 2.02 regardless the inner cladding thickness. As a result of the theoretical investigation, along with the consideration on the fiber fabrication ease, the optimum parameters of the *W*-type two-mode fiber has been determined as  $a/b = 0.6$ ,  $\rho = 1.8$ ,  $\alpha = 2.02$ , and  $V = 6.7$ . Then, core radius and inner cladding radius are  $12.3\mu\text{m}$  and  $20.5\mu\text{m}$ , respectively, at the wavelength of  $1.3\mu\text{m}$ . In the present paper, discussion is restricted to the case where the optimum operating  $V$ -value  $V_0$  is chosen within a theoretical two-mode region. However, the previous design consideration for the two-layer index profile [4] has shown that the operating  $V$ -value is extended up to the effective cutoff  $V$ -value [7] of the  $LP_{21}$  mode. The same idea may be applied to further extension of the optimum operating  $V$ -value for the present *W*-type index profile. If the structural parameters are determined using the idea, this will result in the improvements of  $B_v$  and  $\alpha$  deviation tolerance. This future problem will be appropriately solved on the basis of the experimentally confirmed effective cutoff  $V$ -value of the  $LP_{21}$  mode for the *W*-type index profile.

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